Functional Quantum Theory of the Nonlinear Spinorfield as a Lepton-Hadron Model with Quark-Confinement. II

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The nonlinear spinorfield with Heisenberg's dipole regularization is interpreted as a lepton-quark system. In Section 1, a nonlinear spinorfield equation for two lepton-quark generations is derived by physical arguments and the Heisenberg dipole regularization is formulated for this model. In Section 2, corresponding dynamical equations for local fermion and local boson states are deduced by means of functional quantum theory and their properties are discussed. In Section 3, with this formulation the lowest order S-matrix of elastic lepton-lepton scattering is calculated and is shown to be identical with the results of the corresponding process in quantum electrodynamics resp. weak interactions. The coupling of the local bosons to local fermions is universal, i.e. independent of special lepton or quark representations. Numerical calculations of local boson masses and coupling constants will be given in subsequent papers.

Functional quantum theory is a new formulation of quantum theory and a new field theoretic calculational procedure which allows the treatment of quantized fields with positive metric as well as with indefinite metric beyond perturbation theory. It was developed by Stumpf and coworkers, cf. Stumpf [1]. In particular, it was devoted to the evaluation of a consistent calculational formalism for Heisenberg's nonlinear spinor equation with dipole ghost regularization, cf. Heisenberg [2]. The basic idea of this approach is to regularize the non-renormalizable nonlinear spinorfield by a combination of a real physical particle, a monopole ghost particle and a dipole ghost particle in the one-particle sector of the corresponding state space. For the theoretical evaluation of this model by means of functional quantum theory and for a proper physical interpretation of the corresponding calculational results, the physical meaning of the real particles and of the dipole ghost particles must be uniquely specified. A physical identification of the monopole ghost particles is not needed, since these particles have a vanishing norm in the physical state space. In the original version of Heisenberg [2], the real (physical) one-particle states were assumed to represent nucleons, while the leptons were sometimes identified with the dipole ghosts, cf. Heisenberg [2], Dürr [3]. The reversed role of hadrons and leptons was first proposed by Saller [4], who considered leptons as the real (physical) particles and quarks as dipole ghost particles. Both versions, however, were only vaguely formulated and no hint was given with respect to an analytical evaluation of the idea. In particular, no attempt was made to establish separate dynamical laws for both kinds of particles with quark confinement. In a recent note Stumpf [5] formulated an analytical method for the derivation of a lepton-quark dynamics from the fundamental spinorfield including quark confinement. For simplicity this method was applied to the case of one lepton-quark pair, which is not realistic. In this paper a spinorfield model for two lepton-quark generations is formulated and some elementary conclusions are drawn by the application of the above-mentioned method in the framework of functional quantum theory. In particular, it is shown that first order lepton-lepton scattering in the functional quantum theory of the nonlinear spinorfield leads to the usual first order scattering formula of lepton-boson coupling theories. In contrast to these theories, however, the coupling constants and boson masses are calculable and have not to be given a priori. Numerical evaluation of the results obtained will be given in subsequent papers. In Sect. 1 the nonlinear spinorfield equation for two lepton-quark generations is derived by general physical arguments, while in Sect. 2 and 3 the local fermion and boson solutions and lepton-lepton scattering is discussed. The derivation of the corresponding dynamical equations etc. in the latter sections is strictly deductive and based on functional quantum theory.

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However, the corresponding formalism will not be repeated here. Rather we refer to Stumpf [1] (and preceding papers) whill will be denoted in the following by I, and Stumpf [5], denoted in the following by II.

1. Nonlinear Spinorfield Equation

As mentioned above, we formulate the spinorfield model for two lepton-quark generations. The existence of two or more such generations is a theoretical problem itself, which we will not treat in this context. In addition, we do not introduce colors and Cabibbo-angles, since also these quantities need a theoretical justification which is not available at present.

A spinorfield operator, which allows the description of two generations, is given by $\psi^i_{\alpha k}(x)$, where $i=1,2$ means the generation index, $k=1,2$ the electro-weak isospin index, and $\alpha=1,2,3,4$ the spinor index. Owing to the fact that the spinorfield operators $\psi^i_{\alpha k}(x)$ simultaneously contain lepton-quark pairs, the electro-weak isospin must be universal for leptons and quarks. This result corresponds to the result of an analysis of gauge theories, cf. Rosen [6]. Hence for the formulation of the fundamental spinorfield interaction term, we have to imitate only the electro-weak interactions. We use the CVC-hypothesis, cf. Cheng and O'Neill [7]. According to this hypothesis the electro-weak current for the i-th generation is given by

$$J^{i\mu}(x) := \bar{\psi}^i_{\alpha k}(x)\,\gamma^\mu_{\alpha\beta}\,(\tfrac{1}{2}\,\delta_{kr} + \tfrac{1}{2}\,\vec{\tau}_{kr})\,\psi^i_{\beta r}(x)\,. \quad (1.1)$$

The total current of the two generations then reads

$$J^\mu(x) = \sum_{i=1}^{2} J^{i\mu}(x) \quad (1.2)$$

and we assume for the spinorfield the Lagrangian density

$$\begin{aligned}\mathscr{L}(x) := &-\tfrac{1}{4}\,[\bar{\psi}^i_{\alpha k}(x)\,\gamma^\mu_{\alpha\beta}\,\partial_\mu\,\psi^i_{\beta k}(x)]\\ &+\tfrac{1}{4}\,[\partial_\mu\,\bar{\psi}^i_{\alpha k}(x)\,\gamma^\mu_{\alpha\beta}\,\psi^i_{\beta k}(x)]\\ &+J^\mu(x)\,J_\mu(x)\,,\end{aligned} \quad (1.3)$$

where always the summation convention is used. In accordance with parity symmetric gauge theories, cf. Mohapatra and Sidhu [8], the Lagrangian density (1.3) is parity symmetric and we assume that parity violation results from a spontaneous breakdown of parity symmetry.

In order to simplify the functional formulation it is convenient to define a superspinor by putting $\psi^i_{\alpha k} =: \Psi^i_{1\alpha k}$ and $\bar{\psi}^i_{\alpha k} =: \Psi^i_{2\alpha k}$. With these definitions (1.3) can be rewritten in the following form

$$\begin{aligned}\mathscr{L}(x) = &-\tfrac{1}{4}\,[\Psi_{a_1}(x)\,G^\mu_{a_1a_2}\,\partial_\mu\,\Psi_{a_2}(x)]\\ &+\tfrac{1}{4}\,[\partial_\mu\,\Psi_{a_1}(x)\,G^\mu_{a_1a_2}\,\Psi_{a_2}(x)]\\ &+V_{a_1a_2a_3a_4}\,\Psi_{a_1}(x)\,\Psi_{a_2}(x)\,\Psi_{a_3}(x)\,\Psi_{a_4}(x)\end{aligned} \quad (1.4)$$

where $a_j \equiv i_j, k_j, \varkappa_j, \alpha_j$, $j=1,2,3,4$ and where the spin-tensors G^μ and V are given by the definitions

$$G^\mu_{a_1a_2} := \delta_{i_1i_2}\,\delta_{k_1k_2}\,\lambda_{\varkappa_1\varkappa_2}\,\gamma^\mu_{\alpha_1\alpha_2}\,, \quad (1.5)$$

$$\begin{aligned}V_{a_1a_2a_3a_4} := &\,\delta_{i_1i_2}\,\delta_{i_3i_4}\,\tfrac{1}{4}\,\tau^\nu_{k_1k_2}\,\tau^\nu_{k_3k_4}\\ &\cdot\lambda_{\varkappa_1\varkappa_2}\,\lambda_{\varkappa_3\varkappa_4}\,\gamma^\mu_{\alpha_1\alpha_2}\,\gamma_{\alpha_3\alpha_4\mu}\end{aligned} \quad (1.6)$$

with $\lambda := \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$ and $\tau^\nu_{kh} := (\delta_{kh}, \vec{\tau}_{kh})$.

For the conventional canonical quantization of the spinorfield, (1.3) resp. (1.4) leads to a non-renormalizable spinorfield theory. To avoid this difficulty, Heisenberg [2] proposed a non-canonical quantization. In this type of quantization it is assumed that all anticommutators of the spinorfield vanish at equal times, but that nevertheless a non-trivial spinorfield propagator exists. In consequence of this assumption, the canonical formalism breaks down and the common field theoretical calculational procedures cannot be applied for the evaluation of such a theory, cf. I. In addition, indefinite metric occurs. For the treatment of these types of theories functional quantum theory was developed.

Concerning the selfconsistency of this procedure, it can be shown that coupled linear spinorfield equations for three fields ψ_i, $i=1,2,3$, which lead to a higher order linear field equation for ψ_3, accurately exhibit the non-canonical property for ψ_3 alone if the canonical formalism is applied (canonical embedding), cf. Nagy [9] and I. Dürr [10] studied the canonical embedding of corresponding nonlinear equations, and Ferge [11] and Pouradjam [12] performed perturbation calculations with the nonlinear equations. However, a detailed analysis shows that for non-perturbative calculations in the case of canonical embedding divergencies again occur. As the spinor theory has to be evaluated beyond a perturbation approach, the canonical embedding cannot be applied, i.e. we have to consider the Ψ-field as the analogon of the ψ_3-field and to treat this field without auxiliary fields. Therefore we postulate

$$[\Psi_{a_1}(x)\Psi_{a_2}(y)]_+ = 0 \quad \text{for} \quad x_0 = y_0 . \tag{1.7}$$

Anticommutators between $\overline{\Psi}$ and Ψ are not required as the formation of $\overline{\Psi}$ changes only the components of Ψ. In addition, we need the propagator of the Ψ-field, which we assume to be given by

$$\begin{aligned} F_{a_1 a_2}(x_1 - x_2) &:= \langle 0 | \, T \Psi^{i_1}_{k_1}(x_1) \Psi^{i_2}_{k_2}(x_2) \, | 0 \rangle \\ &= (2\pi)^{-4} \int [(i\gamma^\mu p_\mu + \mu^{i_1}_{k_1})^2 (i\gamma^\mu p_\mu + m^{i_1}_{k_1})]^{-1} \\ &\quad \cdot \delta_{i_1 i_2} \delta_{k_1 k_2} \Lambda \, e^{ip(x_1 - x_2)} \, \mathrm{d}^4 p . \end{aligned} \tag{1.8}$$

In this equation the spinor-indices and the superspinor-indices are suppressed for brevity and

$$\Lambda := (\lambda - \lambda^{\mathrm{T}}) .$$

The masses μ^j_k, m^j_k, $k, j = 1, 2$ are the masses of the two lepton generations $\{e, \nu_e, \mu, \nu_\mu\}$ resp. two quark generations $\{d, u, s, c\}$. This propagator breaks the scale invariance, the electro-weak isospin invariance and the generation invariance of the fundamental Lagrangian (1.4). One can speculate that a kind of Higg's mechanism could be responsible for this symmetry breaking. The simplest explanation, however, could be that the selfconstistent calculation of (1.8) by means of functional quantum theory not only leads to symmetry preserving, but also to symmetry breaking propagators due to the nonlinearity of the corresponding equations for F. In this paper we do not try to discuss this problem. Rather we assume the selfconsistency of the assumptions (1.4), (1.7) and (1.8) and investigate their consequences.

Owing to (1.7) and the rejection of canonical embedding, the common canonical formalism cannot be applied to the Lagrangian (1.4). Hence (1.4) may only be used for a formal derivation of the field equations, while all other conclusions must be drawn from these equations. Field equations can be derived from (1.4) by the usual Lagrangian formalism if it is observed that the functional derivatives of the Lagrangian have to be taken with respect to anticommuting quantities. In this case we obtain from (1.4) the field equation

$$\begin{aligned} &\tfrac{1}{2}(G^\mu_{a_1 a_2} - G^{\mu\mathrm{T}}_{a_1 a_2}) \, \partial_\mu \Psi_{a_2}(x) \\ &= [V_{a_1 a_2 a_3 a_4} - V_{a_2 a_1 a_3 a_4}] \Psi_{a_2}(x) \Psi_{a_3}(x) \Psi_{a_4}(x) . \end{aligned} \tag{1.9}$$

The evaluation of physical quantities comparable with experiment has to be based on (1.7), (1.8) and (1.9) and is the topic of functional quantum theory.

2. Local Fermion and Boson States

The formalism of functional quantum theory shows that local as well as nonlocal fermion and boson states must occur if it is applied to the spinorfield model in Section 1. The local fermion states correspond to the pointlike leptons and quarks, while the local boson states correspond to the bosons which occur in the gauge theories of electro-weak (and strong ?) interactions. For the development of the spinorfield theory it is of special interest to relate its results to those obtained in gauge theories. Hence we first treat the local boson and fermion states.

According to functional quantum theory we introduce the state functionals

$$\begin{aligned} |\mathscr{T}(j,a)\rangle &:= \sum_{n=1}^{\infty} \int \langle 0 | \, T\Psi_{a_1}(x_1) \ldots \Psi_{a_n}(x_n) \, | a \rangle \\ &\quad \cdot | D(\overset{a_1}{x_1} \ldots \overset{a_n}{x_n}) \rangle \, \mathrm{d}^4 x_1 \ldots \mathrm{d}^4 x_n \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} |\mathscr{F}(j,a)\rangle &:= \exp[-\tfrac{1}{2} \int j_{a_1}(x) F_{a_1 a_2}(x_1 - x_2) \\ &\quad \cdot j_{a_2}(x_2) \, \mathrm{d}^4 x_1 \, \mathrm{d}^4 x_2] \, |\mathscr{T}(j,a)\rangle . \end{aligned} \tag{2.2}$$

For these state functionals corresponding functional equations can be derived, cf. I. For actual state calculations such functional equations cannot be directly used. Rather they have to be transformed into functional channel equations in order to incorporate the appropriate boundary conditions for the state functionals. This formalism is discussed in detail in I and preceding papers and we do not repeat this here, but give only the results. By projection operators

$$\begin{aligned} P_k &:= k! \int | D_k(x_1' \ldots x_k') \rangle \\ &\quad \cdot \langle D_k(x_1' \ldots x_k') | \, \mathrm{d}^4 x_1' \ldots \mathrm{d}^4 x_k' \end{aligned} \tag{2.3}$$

the state functional (2.2) can be decomposed into

$$|\mathscr{F}(j,a)\rangle = \sum_{l=0}^{\infty} |\mathscr{F}_{\lambda+l}(j,a)\rangle \tag{2.4}$$

and

$$|\mathscr{F}_k(j,a)\rangle := P_k \, |\mathscr{F}(j,a)\rangle , \tag{2.5}$$

where $|\mathscr{F}_\lambda(j,a)\rangle$ is the lowest non-trivial contribution of (2.2) to the functional state $|\mathscr{F}(j,a)\rangle$. For $|\lambda a\rangle \equiv |\mathscr{F}_\lambda(j,a)\rangle$ an effective channel equation can be derived, which reads in a second order approximation

$$A_{1a_1}(x)\,|\,\lambda\rangle + \hat{V}_{a_1a_2a_3a_4} d_{a_2}(x)\, d_{a_3}(x) \int G_{a_4b}(x,x')\, C_b(x')\, d^4x'\,|\,\lambda\rangle + \hat{V}_{a_1a_2a_3a_4} d_{a_2}(x)\, d_{a_3}(x) \int G_{a_4b_1}(x,x')\, \hat{V}_{b_1b_2b_3b_4} d_{b_2}(x')\, d_{b_3}(x') \cdot G_{b_4c}(x,x'')\, C_c(x'')\, d^4x'\, d^4x''\,|\,\lambda\rangle + \cdots = 0 \,. \tag{2.6}$$

We give only the definition of A and $\hat{V}$ and refer for the other quantities to I. It is

$$A_{1a}(x) := D_{a_1a_2}(x)\,\partial_{a_2}(x) + 3\,\hat{V}_{a_1a_2a_3a_4} \int F_{a_2b}(x-x')\, j_b(x')\, d^4x'\,\partial_{a_3}(x)\,\partial_{a_4}(x) \tag{2.7}$$

and

$$\hat{V}_{a_1a_2a_3a_4} := V_{a_1a_2a_3a_4} - V_{a_2a_1a_3a_4}\,, \tag{2.8}$$

and

$$D_{a_1a_2}(x) := \tfrac{1}{2}(G^{\mu}_{a_1a_2} - G^{\mu\mathrm{T}}_{a_1a_2})\,\partial_\mu \,. \tag{2.9}$$

The local fermion functionals are given by

$$|\,1,f\rangle := \int \varphi_a(x)\, j_a(x)\, d^4x\,|\,D_0\rangle \,. \tag{2.10}$$

In order to obtain non-trivial equations for these states at least the second order contribution in (2.6) must be taken into account. We postpone the direct evaluation of such equations and draw only some general conclusions. Since the functional equations are invariant under space-time translations, this must hold also for the channel equations. Hence the most general form of a local fermion equation has to be

$$\int K_{a_1a_2}(x-x')\,\varphi_{a_2}(x')\, d^4x' = 0\,, \tag{2.11}$$

where $K(x-x')$ is the selfenergy operator of the local fermions and its special expression can be derived from (2.6). The Fourier-transform of (2.11) reads

$$\tilde{K}_{a_1a_2}(p)\,\tilde{\varphi}_{a_2}(p) = 0\,. \tag{2.12}$$

For a momentum eigenstate of a local fermion it is $\tilde{\varphi}_a(p) = \chi_a(P)\,\delta(P-p)$ and (2.12) has to be satisfied with a definite P. Since the local fermion solutions uniquely determine the propagator (1.8), in a selfconsistent theory the propagator must be the Greenfunction of the corresponding eigenvalue Eqs. (2.11), resp. (2.12), cf. I. This condition leads to $\tilde{K}_{a_1a_2}(p)\,\tilde{F}_{a_2a_3}(p) = 1$ in momentum space and from this equation if follows immediately that

$$\tilde{K}_{a_1a_2}(p) = \delta_{i_1i_2}\,\delta_{k_1k_2}\,\Lambda^{-1}(i\gamma^\mu p_\mu + \mu^{i_1}_{k_1})^2 \cdot (i\gamma^\mu p_\mu + m^{i_1}_{k_1})\,. \tag{2.13}$$

Hence the selfconsistency condition determines the fermion selfenergy. Of course in the further evaluation of the theory the selfconsistency has to be proven. From II it follows that the projection on the lepton- resp. quark subspace of local fermions does not change the manifold of solutions of (2.13). Hence in the one-particle sector of local fermion solutions it is sufficient to directly study the spinorfield channel Eq. (2.12) with (2.13).

Concerning the local boson states, these states are special solutions of the general boson states

$$|\,2,b\rangle := \frac{1}{2!}\int \varphi_{a_1a_2}(x_1x_2) \cdot j_{a_1}(x_1)\, j_{a_2}(x_2)\, d^4x_1\, d^4x_2\,|\,D_0\rangle \,. \tag{2.14}$$

To obtain the corresponding eigenvalue equation, we symmetrize (2.6) by multiplying it with

$$\int j_b(x')\, s_{ba_1}(x'-x)\, dx'$$

and integrating over x. In the resulting equation the selfenergy terms with respect to the coordinates x_1 resp. x_2 can be separated from the interaction terms between x_1 and x_2 by a method described in I and preceding papers. The selfenergy terms with respect to x_1 resp. x_2 must be equal to the fermion selfenergies K referred to x_1 resp. x_2, as these terms just occur in one coordinate, cf. I. We do not discuss the derivation here in detail but only give the result. If the interaction energy in (2.6) is taken into account in the lowest order, we obtain for the boson state functionals the equation

$$\begin{aligned}&\int s_{a_1b}(x_1-x_1'')\,K_{ba_1'}(x_1''-x_1')\,\varphi_{a_1'a_2}(x_1'x_2)\, d^4x_1'\, d^4x''\\ &\quad + \int s_{a_2b}(x_2-x_2'')\,K_{ba_2'}(x_2''-x_2')\,\varphi_{a_1a_2'}(x_1x_2')\, d^4x_2'\, d^4x_2''\\ &\quad - 3\int s_{a_1b}(x_1-x_1')\,\hat{V}_{ba_2'a_3'a_4'}\,F_{a_2'a_2}(x_1'-x_2)\,\varphi_{a_3'a_4'}(x_1'x_1')\, d^4x_1'\\ &\quad + 3\int s_{a_2b}(x_2-x_2')\,\hat{V}_{ba_2'a_3'a_4'}\,F_{a_2'a_1}(x_2'-x_1)\,\varphi_{a_3'a_4'}(x_2'x_2')\, d^4x_2' = 0\,.\end{aligned} \tag{2.15}$$

The momentum representation of (2.15) reads

$$\begin{aligned}&\tilde{s}_{a_1b}(s_1)\,\tilde{K}_{ba_1'}(s_1)\,\tilde{\varphi}_{a_1'a_2}(s_1s_2) + \tilde{s}_{a_2b}(s_2)\,\tilde{K}_{ba_2'}(s_2)\,\tilde{\varphi}_{a_1a_2'}(s_1s_2)\\ &\quad - [3\,\tilde{s}_{a_1b}(s_1)\,\hat{V}_{ba_2'a_3'a_4'}\,\tilde{F}_{a_2'a_2}(-s_2) - 3\,\tilde{s}_{a_2b}(s_2)\,\hat{V}_{ba_2'a_3'a_4'}\,\tilde{F}_{a_2'a_1}(-s_1)]\\ &\quad \cdot \int \tilde{\varphi}_{a_3'a_4'}(p_1p_2)\,\delta(p_1+p_2-s_1-s_2)\, d^4p_1\, d^4p_2 = 0\,.\end{aligned} \tag{2.16}$$

According to the selfconsistency condition the selfenergy operator $\tilde{K}$ must have the form (2.13). We choose $\tilde{s}$ to be identical with $\tilde{F}$, i.e. $\tilde{s} \equiv \tilde{F}$ and due to $\tilde{F}\tilde{K} = 1$ we obtain from (2.16) the equation

$$\tilde{\varphi}_{a_1 a_2}(s_1 s_2) - \tilde{W}_{a_1 a_2 a_3' a_4'}(s_1 s_2) \int \tilde{\varphi}_{a_3' a_4'}(p_1 p_2)\, \delta(p_1 + p_2 - s_1 - s_2)\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 = 0\,, \tag{2.17}$$

where the definition

$$\tilde{W}_{a_1 a_2 a_3' a_4'}(s_1 s_2) := \tfrac{3}{2}[\tilde{F}_{a_1 b}(s_1)\, \hat{V}_{b a_2' a_3' a_4'} \tilde{F}_{a_2' a_2}(-s_2) - \tilde{F}_{a_2 b}(s_2)\, \hat{V}_{b a_2 a_3' a_4'} \tilde{F}_{a_2' a_1}(-s_1)] \tag{2.18}$$

is used. With the transformation to the center of mass coordinates

$$s_1 = \tfrac{1}{2} s_c + s_r\,, \quad p_1 = \tfrac{1}{2} p_c + p_r\,,$$
$$s_2 = \tfrac{1}{2} s_c - s_r\,, \quad p_2 = \tfrac{1}{2} p_c - p_r \tag{2.19}$$

Eq. (2.17) takes the form

$$\varphi_{a_1 a_2}(s_c s_r) - W_{a_1 a_2 a_3' a_4'}(s_c s_r) \int \varphi_{a_3' a_4'}(p_c p_r) \cdot \delta(p_c - s_c)\, \mathrm{d}p_c\, \mathrm{d}p_r = 0\,, \tag{2.20}$$

where here and in the following we omit the tilde. Before we further evaluate (2.17) resp. (2.20), it has to be remarked that as a consequence of $\tilde{s} \equiv \tilde{F}$ an additional inhomogeneous term in the symmetrized functional Eq. (2.6) may occur, which is a solution of the equation $\tilde{\partial}(t)\tilde{K}(t)|\lambda\rangle^0 = 0$ and leads to scattering states.

We first consider the homogeneous Eqs. (2.17) resp. (2.20) for bound states. For the bound boson states we make the ansatz

$$\varphi_{a_1 a_2}(s_c s_r) = \delta(P - s_c)\, \chi_{a_1 a_2}(s_r) \tag{2.21}$$

and obtain by substitution into (2.20) and subsequent elimination of the δ-distribution the equation

$$\chi_{a_1 a_2}(s_r) - W_{a_1 a_2 a_3' a_4'}(P, s_r) \cdot \int \chi_{a_3' a_4'}(p_r)\, \mathrm{d}^4 p_r = 0\,. \tag{2.22}$$

With the abbreviation

$$\boldsymbol{\chi} := \chi_{a_1 a_2} := \int \chi_{a_1 a_2}(p_r)\, \mathrm{d}^4 p_r \tag{2.23}$$

the eigenvalues of Eq. (2.22) follow from the equation

$$[1 - \int \mathbf{W}(P, s_r)\, \mathrm{d}^4 s_r]\, \boldsymbol{\chi} = 0\,, \tag{2.24}$$

where the fat capitals etc. denote the spintensors. If we perform according to II the transition to lepton, quark or mixed representations we have to put for wave functions of the type (2.21)

$$\boldsymbol{\chi}(s_r) = \mathbf{F}_A(\tfrac{1}{2} P + s_r)\, \mathbf{F}_B(\tfrac{1}{2} P - s_r)\, \boldsymbol{\chi}_{AB}(s_r)\,. \tag{2.25}$$

From this relation it follows that the eigenvalue Eq. (2.24) is not changed by the transition to the various representations. This means that the local boson states are universal with respect to quark and lepton representations. The terminus local boson stems from the fact, that eigenvalues depend only on (2.23), i.e. on $\chi_{a_1 a_2}(x, x)$ in coordinate space.

Summarizing the results of this section, we have shown that the local fermions as well as the local bosons are universal, i.e. independent of quark or lepton representations. Then one may expect that the coupling between local fermions and local bosons must also be universal. From this it follows that the local bosons should be interpreted as the carriers of electro-weak interactions which are universal for leptons and quarks. In order to clarify this problem we study scattering processes between local fermions and local bosons.

3. Local Fermion-Fermion Scattering

The simplest scattering process that can be treated in our model, is the scattering between local fermions. For this process we may use Eq. (2.17) with an inhomogeneous term $\boldsymbol{\varphi}^0(s_1 s_2)$ which arise from the symmetrizing procedure, as already mentioned in Section 2. This leads to

$$\begin{aligned} \boldsymbol{\varphi}(s_1 s_2) - \mathbf{W}(s_1 s_2) \int \boldsymbol{\varphi}(p_1 p_2) &\cdot \delta(p_1 + p_2 - s_1 - s_2)\, \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \\ &= \tfrac{1}{2}[\boldsymbol{\varphi}_A^0(s_1)\boldsymbol{\varphi}_B^0(s_2) - \boldsymbol{\varphi}_B^0(s_1)\boldsymbol{\varphi}_A^0(s_2)] \\ &=: \boldsymbol{\varphi}^0(s_1 s_2)\,, \end{aligned} \tag{3.1}$$

where $\boldsymbol{\varphi}^0(s_1 s_2)$ is an antisymmetric solution of the equation

$$\partial(t)\, \mathbf{K}(t) \int \boldsymbol{\varphi}^0(s_1 s_2)\, j(s_1)\, j(s_2)\, \mathrm{d}^4 s_1\, \mathrm{d}^4 s_2\, |D_0\rangle = 0 \tag{3.2}$$

with two ingoing resp. outgoing free local fermion states. With center of mass coordinates (2.19) equation (3.1) reads

$$\boldsymbol{\varphi}(s_c s_r) - \mathbf{W}(s_c, s_r) \int \boldsymbol{\varphi}(s_c, p_r)\, \mathrm{d}^4 p_r = \boldsymbol{\varphi}^0(s_c s_r)\,. \tag{3.3}$$

By means of the definitions

$$\boldsymbol{\Phi}(s_c) := \int \boldsymbol{\varphi}(s_c, p_r)\, \mathrm{d}^4 p_r\,;$$
$$\boldsymbol{\Phi}^0(s_c) := \int \boldsymbol{\varphi}^0(s_c, p_r)\, \mathrm{d}^4 p_r \tag{3.4}$$

and

$$\mathbf{M}(s_c) := -\int \mathbf{W}(s_c, s_r)\, d^4 s_r \qquad (3.5)$$

the exact solution of (3.3) is straightforward and gives the result

$$\boldsymbol{\varphi}(s_c s_r) = \mathbf{W}(s_c s_r)[1 + \mathbf{M}(s_c)]^{-1} \boldsymbol{\Phi}^0(s_c) + \boldsymbol{\varphi}^0(s_c s_r). \qquad (3.6)$$

The operator $[1 + \mathbf{M}(s_c)]^{-1}$ is the Greenfunction of the local boson eigenvalue equation (2.24). The solutions of (2.24) must be Poincaré-invariant and may have different eigenvalues for different symmetries. The various symmetries may be classified by the index α with the corresponding spintensor $\boldsymbol{\chi}^\alpha$ and a corresponding boson mass eigenvalue $m_{B\alpha}$. Then a general $\boldsymbol{\chi}$ can be expressed by a linear combination of the $\boldsymbol{\chi}^\alpha$, i.e. $\boldsymbol{\chi} = \sum_\alpha c_\alpha \boldsymbol{\chi}^\alpha$. Substitution of this expansion into (2.22) gives

$$[1 + \mathbf{M}(s_c)] \sum c_\alpha \boldsymbol{\chi}^\alpha = 0. \qquad (3.7)$$

Owing to their symmetry properties the $\boldsymbol{\chi}^\alpha$ must be orthonormal in spinspace. Furthermore, the projection of $\mathbf{M}(s_c)$ on the set of solutions $\{\boldsymbol{\chi}^\alpha\}$ must be diagonal. Hence by projecting with $\boldsymbol{\chi}^\beta$ from the left on (3.7) we obtain

$$\delta_{\alpha\beta}[1 + M_{\alpha\alpha}(s_c)]\, c_\alpha = 0 \qquad (3.8)$$

and due to the Poincaré-invariance of the eigenvalues, this equation must have the form

$$\delta_{\alpha\beta}\, g_\alpha(s_c)(s_c^2 - m_{B\alpha}^2)\, c_\alpha = 0, \qquad (3.9)$$

where $g(s_c)$ is an entire function of s_c without zeros. From this it follows immediately that the Greenfunction of (2.22) must have the general form

$$[1 + \mathbf{M}(s_c)]^{-1} = \sum_\alpha \boldsymbol{\chi}^\alpha \otimes \boldsymbol{\chi}^\alpha \frac{g_\alpha^{-1}(s_c)}{(s_c^2 - m_{B\alpha}^2)} =: \sum_\alpha \mathbf{D}_\alpha(s_c)\, g_\alpha^{-1}(s_c) \qquad (3.10)$$

i.e. a weighted sum of boson propagators. Hence by means of (3.10) the general solution (3.6) can be expressed in the two-particle coordinates s_1, s_2 by

$$\boldsymbol{\varphi}(s_1 s_2) = \mathbf{W}(s_1 s_2) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2) \cdot \boldsymbol{\Phi}^0(s_1 + s_2) + \boldsymbol{\varphi}^0(s_1 s_2). \qquad (3.11)$$

According to II from this spinorfield amplitude the lepton- resp. quark amplitudes have to be derived by projections which read in momentum space

$$\boldsymbol{\varphi}(s_1 s_2) = \mathbf{F}_A(s_1)\, \mathbf{F}_B(s_2)\, \boldsymbol{\chi}_{AB}(s_1 s_2). \qquad (3.12)$$

In general this projection has to be applied before an integration of the corresponding channel equation has been performed, since the projectors F_X modify the polestructure of the kernel of the integral channel equation. However, in the simple case of a local interaction such as (2.20), such a modification does not take place and hence the projection can be applied equally well after the integration, i.e. to the final formula (3.11). This gives for instance for lepton-lepton scattering

$$\boldsymbol{\chi}_{ll}(s_1 s_2) = \mathbf{F}_q^{-1}(s_1)\, \mathbf{F}_q^{-1}(s_2)\, \mathbf{W}(s_1 s_2) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2)\, g_\alpha^{-1}(s_1 + s_2)\, \boldsymbol{\Phi}^0(s_1 + s_2) + \mathbf{F}_q^{-1}(s_1)\, \mathbf{F}_q^{-1}(s_2)\, \boldsymbol{\varphi}^0(s_1 s_2). \qquad (3.13)$$

We now assume that two leptons with momentum k_1, k_2 are the ingoing particles. Then we have

$$\boldsymbol{\varphi}^0(s_1 s_2) := \tfrac{1}{2}(2\pi)^8 C_0(k_1 k_2)[\mathbf{u}(k_1)\, \mathbf{u}(k_2)\, \delta(s_1 - k_1)\, \delta(s_2 - k_2)]_{as}. \qquad (3.14)$$

The constant C_0 has to be chosen in such a way that the corresponding physical ingoing lepton states

$$\boldsymbol{\chi}_{ll}^0(s_1 s_2) = \mathbf{F}_q^{-1}(s_1)\, \mathbf{F}_q^{-1}(s_2)\, \boldsymbol{\varphi}^0(s_1 s_2) \qquad (3.15)$$

are properly normalized, i.e. that

$$\boldsymbol{\chi}_{ll}^0(s_1 s_2) = \tfrac{1}{2}(2\pi)^8 [\mathbf{u}(k_1)\, \mathbf{u}(k_2)\, \delta(s_1 - k_1)\, \delta(s_2 - k_2)]_{as} \qquad (3.16)$$

results. For $\boldsymbol{\Phi}^0(s_1 + s_2)$ we obtain in this way

$$\boldsymbol{\Phi}^0(s_1 + s_2) = \int \boldsymbol{\varphi}^0(p_1 p_2)\, \delta(p_1 + p_2 - s_1 - s_2)\, dp_1\, dp_2 = \tfrac{1}{2}(2\pi)^8 \delta(k_1 + k_2 - s_1 - s_2)[\mathbf{u}(k_1)\, \mathbf{u}(k_2)]_{as}. \qquad (3.17)$$

Owing to this assumption the solution (3.13) depends on k_1 and k_2, i.e. $\boldsymbol{\chi}_{ll}(s_1 s_2) \equiv \boldsymbol{\chi}_{ll}(s_1 s_2, k_1 k_2)$. If two outgoing leptons with momenta k_1' and k_2' are considered, according to the method of functional quantum theory the S-matrix is given by the scalarproduct

$$S(k_1' k_2', k_1 k_2) := \int \boldsymbol{\sigma}_l(s_1 k_1') \boldsymbol{\sigma}_l(s_2 k_2') \boldsymbol{\chi}_{ll}(s_1 s_2, k_1 k_2) \, \mathrm{d}^4 s_1 \, \mathrm{d}^4 s_2 , \tag{3.18}$$

where $\boldsymbol{\sigma}_l(s_i k_i')$, $i = 1, 2$ are the contravariant one-particle lepton eigenfunctions, cf. I. The contravariant one-particle eigenfunctions have to be determined by the conditions

$$\int \boldsymbol{\sigma}_A(p) \boldsymbol{\varphi}_B(p) \, \mathrm{d}^4 p = g_{AB} , \tag{3.19}$$

where g_{AB} is the metrical tensor of the pointlike one-particle states. However, these conditions do not completely determine $\boldsymbol{\sigma}_A(p)$. In particular, with respect to the p_0 component there is some arbitrariness. It can be shown, for instance, that for lepton states the ansatz

$$\boldsymbol{\sigma}_l(p) = (2\pi)^3 \delta^3(\mathfrak{k} - \mathfrak{p}) \, \omega(m) \, e^{-\sigma[p_0 - \omega(m)]^2} \, \mathbf{u}(k) \tag{3.20}$$

in the limit of positive infinite σ satisfies all conditions. Substitution of (3.17) into (3.13) and of (3.13) into (3.18) then gives with (3.20)

$$S(k_1' k_2', k_1 k_2) = \mathbf{1} + R(k_1' k_2', k_1 k_2) , \tag{3.21}$$

where R is defined by

$$\begin{aligned} R := \int & \mathrm{d}^4 s_1 \, \mathrm{d}^4 s_2 \, \mathbf{u}(k_1') \, \mathbf{u}(k_2') \, \delta(\mathfrak{k}_1' - \mathfrak{s}_1) \, \omega_1 e^{-\sigma(s_1^0 - \omega_1)^2} \delta(\mathfrak{k}_2' - \mathfrak{s}_2) \, \omega_2 e^{-\sigma(s_2^0 - \omega_2)^2} \\ & \cdot \mathbf{F}_q^{-1}(s_1) \mathbf{F}_q^{-1}(s_2) \mathbf{W}(s_1 s_2) \sum_\alpha \mathbf{D}_\alpha(s_1 + s_2) g_\alpha^{-1}(s_1 + s_2) \, \delta(k_1 + k_2 - s_1 - s_2) [\mathbf{u}(k_1) \mathbf{u}(k_2)]_{\mathrm{as}} . \end{aligned} \tag{3.22}$$

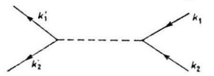
Owing to $F_{ab}(-s) = -F_{ba}(s)$ and $\mathbf{F}_q^{-1}(s) \mathbf{F}(s) = \mathbf{F}_l(s)$ we obtain

$$\mathbf{F}_q^{-1}(s_1) \mathbf{F}_q^{-1}(s_2) \mathbf{W}(s_1 s_2) = \tfrac{3}{2} [\mathbf{F}_l(s_1) \mathbf{F}_l(s_2) \hat{\mathbf{V}}]_{\mathrm{as}} . \tag{3.23}$$

After performing the integration over $\mathrm{d}^3 s_1$ and $\mathrm{d}^3 s_2$ in (3.22), the residual integration over s_1^0 and s_2^0 remains. With $\mathfrak{s}_1 \equiv \mathfrak{k}_1$ and $\mathfrak{s}_2 \equiv \mathfrak{k}_2$ the corresponding poles of $\mathbf{F}_l(s_1)$ resp. $\mathbf{F}_l(s_2)$ are $s_1^0 = \pm \omega_1$, $s_2^0 = \pm \omega_2$. In the limit of positive infinite σ only the poles ω_1 and ω_2 contribute to (3.22), i.e. we obtain $s_1 \equiv k_1'$, $s_2 \equiv k_2'$, and R takes the form

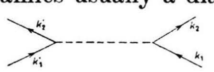
$$R = \mathbf{u}(k_1') \mathbf{u}(k_2') [\hat{\mathbf{V}}]_{\mathrm{as}} \sum_\alpha \mathbf{D}_\alpha(k_1' + k_2') g_\alpha^{-1}(k_1' + k_2') \, \delta(k_1 + k_2 - k_1' - k_2') [\mathbf{u}(k_1) \mathbf{u}(k_2)]_{\mathrm{as}} . \tag{3.24}$$

This expression of the R-matrix corresponds to a diagram of the type



(a)

In quantum electrodynamics usually a diagram of the type



(b)

for lepton-lepton scattering is considered, where in comparison with (a) the direction of k_2 and k_2' is reversed. If this reversion of the direction is taken into account in (3.24) by replacing k_2 and k_2' by $-k_2$ and $-k_2'$, then (3.24) has the same structural form as the corresponding expression in quantum electrodynamics. In contrast to quantum electrodynamics the expression (3.24) is more general, as several types of bosons occur. For these local boson-local fermion coupling it is remarkable that the coupling constants are universal, i.e. do not depend on the special mass etc. of the fermions, as $g^{-1}(P)$ depends only on the total energy of the exchange boson.

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